

Wave–particle interaction

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Abstract. We seek a description of plasma wave–particle interactions in the weakly collisional regime. Because weak collisions produce a qualitative change in the plasma degrees of freedom without totally suppressing kinetic effects, neither the Vlasov limit nor the fluid moment limit are found to be an adequate description of experimental data. Illustrative examples of data that require a weakly collisional description are discussed.

1. Introduction

Every discussion of plasma wave–particle interaction must contain assumptions about the plasma degrees of freedom. Implied in the term is the existence of wave and particle degrees of freedom that are separable to a first approximation and that interact through specific mechanisms. Typically this separation is accomplished conceptually through the use of hierarchy expansions, weak turbulence, and quasilinear theory. To describe a given experiment it is frequently possible to select one or two wave dispersion relations which appear to be relevant to the wave activity and to use the electron and ion temperatures and flow velocities to describe the particle degrees of freedom. However, this separation of waves and particles is more a separation of time scales than of particular variables, because perturbations of both the temperatures and the flow velocities may be central to wave propagation. Furthermore, strictly speaking in the Vlasov approximation, there are no particles. There are two aspects of plasma dynamics that complicate the program of separating the wave and particle degrees of freedom. One is nonlinear effects that may couple waves and particles strongly, and the other is that the use of a small number of dispersion relations is not always justified. We reconsider the question of wave–particle interaction in the context of recent experiments on electrostatic ion waves. Both nonlinear effects associated with plasma wave–particle resonance and the question of the linear excitation spectrum are addressed. In weakly coupled plasmas, new dispersion relations are observed, related to the Case Van Kampen continuum of the Vlasov approximation. These new modes are necessary in order to achieve an adequate description of observed wave dynamics. After a review of the theoretical context, we describe the experimental configuration of the experiment and present results of measurements of the plasma response function. The issues raised by these experiments for the description of wave–wave interaction as well as wave–particle interaction will be summarized and prospects for new experiments described.

2. Summary of theory

Weakly coupled (or weakly collisional) plasma is a remarkable many-body system where the individual particles interact almost exclusively through the macroscopic mean-fields. The high kinetic energy and mobility of ions and electrons result in the electromagnetic particle–particle interaction fields being mostly screened out. To a first approximation, particle trajectories are determined by the effects of the specific residual macroscopic electric and magnetic mean-fields that escape the shielding effect. An important class of such unshielded fields can be described as plasma waves. The first task of understanding wave–particle interaction is to understand the nature of the mean-field waves which may participate in affecting particle orbits.

Curiously, the fact that the plasma consists of discrete particles is irrelevant in the approximation of weak coupling because it is only the large scale currents and densities that are responsible for the mean fields. Concretely, the weak-coupling approximation consists of factorizing the probability density for the many-body plasma system into a product of one-body distribution functions for each particle of each species [1].

$$f^{(N)}(\vec{x}_{e1}, \vec{v}_{e1}, \dots, \vec{x}_{eN}, \vec{v}_{eN}, \vec{x}_{i1}, \vec{v}_{i1}, \dots, \vec{x}_{iN}, \vec{v}_{iN}) = \prod_{k=1}^N f^{(1)}(\vec{x}_{ek}, \vec{v}_{ek}) f^{(1)}(\vec{x}_{ik}, \vec{v}_{ik}). \quad (1)$$

Here we have assumed an equal and constant number (N) of electrons and a single species of ions. A one-body distribution $f^{(1)}(\vec{x}, \vec{v})$ defines the probability of finding any ion at position \vec{x} and velocity \vec{v} . The factorization means that there is only a single one-body distribution function that must be determined for each kind of particle. In this approximation where the one-body distribution for each particle species obeys an equation that describes conservation of probability in the particle phase-space under the action of the mean fields particle discreteness effects are ignored completely.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \frac{q}{m} \left(E + \frac{\vec{v}}{c} \times B \right) \cdot \nabla_v f = 0. \quad (2)$$

Although particle discreteness is lost in the Vlasov approximation, it is possible to approach equation (2) from either a Lagrangian (the particle orbits are the characteristics of equation (2)) or an Eulerian (phase-space fluid) point of view. Another way to write the Vlasov equation employs a Poisson bracket with the Hamiltonian function H that defines the particle orbits given the plasma mean-fields

$$\frac{\partial f}{\partial t} + [f, H] = 0. \quad (3)$$

Self consistency and closure of the linear description of plasma dynamics is obtained by using Maxwell's equations using the lowest moments of the one-particle probability function to determine the plasma densities and currents.

Plasma dynamics in the Vlasov approximation is very subtle. Although dispersion relations for collective modes are frequently used in this limit, in reality Vlasov plasma has a continuum of degrees of freedom and has no dispersion relations in the usual sense [2]. Rather than to speak of wave–particle interactions (where there is no particle discreteness) one should probably speak of the dynamics of the continuum [3]. Even wave absorption in inhomogeneous plasma can be thought of as a kind of mode conversion process in the full wave phase-space which is a continuum [4].

However, the effects of particle discreteness are not absent in plasmas, and phenomena such as Coulomb collisions are often important. Even fairly simple corrections to the Vlasov picture which incorporate the effects of particle discreteness and correlations have

a qualitative effect on the degrees of freedom and result in the continuum breaking up into discrete dispersion relations among which are the usual hot plasma dispersion relations found in textbooks [5, 6]. Depending on the collision operator (for example the Landau collision integral), it may be that a continuum at high phase velocity coexists with discrete modes at low phase velocity. Because the introduction of particle discreteness is a singular perturbation, the limit of weak coupling is different from the Vlasov approximation. Usually, dispersion relations are obtained by using Maxwell's equations to describe mean-field waves with the lowest moments of the perturbed one-body distribution function defining the currents and densities. Typically waves may be described by a low number of moments even including the effects of Landau damping [7]. However, the complete space of the plasma linear degrees of freedom is found by using the kinetic equation as a dynamical equation together with Maxwell's equations.

For concreteness, and for comparison with the experiment we will consider electrostatic ion waves (electrostatic waves below the ion plasma frequency) where the plasma response is quasineutral and the electron response is adiabatic. The one-body ion distribution is expanded (linearized) according to the smallness of the perturbation amplitude ($f^{(1)} = f_0 + f_1 + \dots$). In this case we can describe the relevant linear plasma degrees of freedom through the ion kinetic equation

$$\frac{\partial f_1}{\partial t} + v_{\parallel} \frac{\partial f_1}{\partial z} - \Omega_{ci} \frac{\partial f_1}{\partial \varphi} - \frac{c_s^2}{n_0} \nabla_x n_1 \cdot \nabla_v f_0 = C(f_0, f_1). \quad (4)$$

Here we define the gyrophase φ for the particle motion of ions about the fixed magnetic field with nominal frequency Ω_{ci} and velocity parallel to the magnetic field of v_{\parallel} (z -axis). The ion acoustic velocity is defined in terms of the electron temperature and the ion mass by $c_s^2 = T_e/m_i$, and the electron response is taken to be adiabatic ($n_{e1}/n_{e0} = e\Phi/\kappa T_e$) and quasineutral ($n_{ek} \sim n_{ik}$). The ion density of each order is defined by an integral over the corresponding distribution $n_k = \int f_k(\vec{x}, \vec{v}) d^3v$. Inclusion of the collision term results in discrete dispersion relations (for example with a Fokker-Planck operator $C(f_0, f_1) = \beta \nabla_v \cdot [(\vec{v} - \vec{v}) f_1 - v_i^2 \nabla_v f_1]$ with \vec{v} and v_i^2 defined by the first and second moments of the unperturbed distribution, respectively). The collision frequency β is of the order of the plasma frequency divided by the square root of the number of particles in a sphere of radius equal to the Debye-shielding length. Formulation of the linear wave theory without collisions is problematic in that there is a singularity in the linear response function. The natural ansatz for the linear response defined by equation (4) is a function of the form $f_1 = A(k_{\parallel}, \omega, n, v_{\parallel}, v_{\perp}, k_{\perp}) \exp(-i\omega t + ik_{\parallel}z + in\varphi)$. In the absence of particle discreteness effects ($C = 0$) the linear response has singularities at the velocities corresponding to wave-particle resonance $\omega - n\Omega_{ci} = k_{\parallel}v_{\parallel}$. A finite collision frequency β results in a linear theory with non-singular eigenfunctions. The solution of equation (4) in one dimension was performed quite a long time ago as was a solution in three dimensions, but no mention was made of the fact that the degrees of freedom are altered [8, 9]. Indeed, we have found no mention of this fact in the literature. If one defines, for example, a boundary value problem and follows the general approach of Landau, then one finds that the Landau pole usually retained is only slightly modified. However, even for a very small collision frequency, much of the rest of the collisionless spectrum is radically changed. The infinite accumulation of poles near the origin, in particular, is completely removed. Figure 1 shows a typical example which shows the contrast between small β and $\beta = 0$. In addition to the usual dispersion relations of hot plasma, we note that many of the Landau poles are also transformed into true eigenfunctions of the plasma response and new dispersion relations are also obtained. This has important consequences for the experiment as we will discuss later.

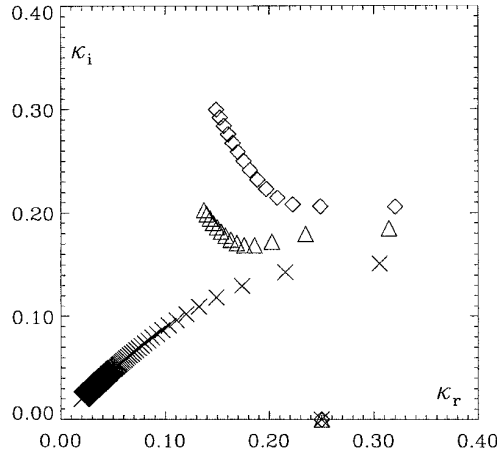


Figure 1. Complex κ -plane for one-dimensional electrostatic ion waves showing the Landau poles (\times) with the infinite accumulation at the origin and the modification that occurs to some of the poles as a function of finite collision frequency $\beta/\omega = 0.03125$ (Δ), and 0.05 (\diamond). Here $\kappa = k\sqrt{2}v_{th}/\omega$ and $v_{th} = \sqrt{T_i/m_i}$.

One problem with the Vlasov limit is that the linear theory is only valid for a time which is short compared with the time it takes for particles in resonance to execute bounce motion

$$\tau_b = \left[\frac{e}{4\pi^2 m_i} J_n \left(\frac{k_{\perp} v_{\perp}}{\Omega_{ci}} \right) k_{\parallel}^2 \Phi_0 \right]^{-1/2}$$

in the wave trough of the electrostatic potential $\Phi = \Phi_0 [\exp(-i\omega t + ik_{\parallel} z + ik_{\perp} x) + \text{c.c.}]/2$. The validity of linear dynamics is further complicated by the fact that bounce motion is disrupted when resonances overlap and particle motion becomes chaotic. The width of a resonance is determined by the maximum relative velocity that results in trapped bounce motion. The velocity width of a resonance at $v = \omega/k_{\parallel} - n\Omega/k_{\parallel}$ is given by

$$\Delta v = 4 \sqrt{\frac{e\Phi_0}{m_i} J_n(k_{\perp} \rho)}$$

but this structure can be lost when orbits are chaotic.

An accounting of energy with finite collision frequency requires, nevertheless, that one develop a quasilinear theory that takes into account the modification of the time averaged distribution in addition to the energy of the wave spectrum. The only new aspect that we note with respect to quasilinear theory is that the parts of the discrete spectrum that are not in textbooks (which we call the kinetic modes) are very sensitive to the particle velocity distribution function. Thus it is more difficult to separate the evolution of the distribution function and of the waves. A similar situation, however, pertains to the ion Bernstein wave which produces ion heating and has a wavelength that depends strongly on the ion temperature.

Important questions concerning the understanding of wave-particle resonance include understanding the degrees of freedom of a plasma as a function of the excitation amplitude as well as the transition between quasilinear and various kinds of nonlinear dynamics. In the next section we will present some experimental data concerning these problems and suggest where future experiments may produce important clarifications.

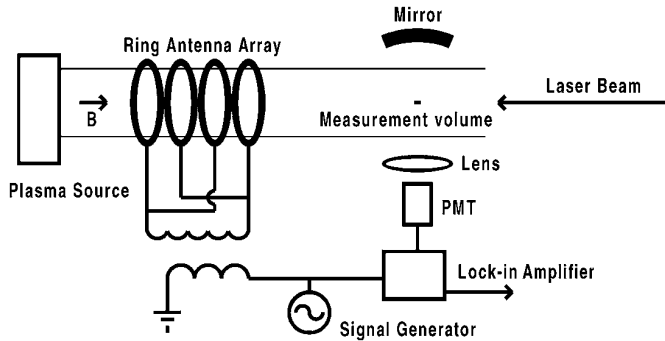


Figure 2. Experimental set-up. Ion motion is Doppler resolved along the magnetic field direction and in all three spatial coordinates.

3. Experimental system

Electrostatic ion cyclotron waves propagating at a small angle relative to a uniform magnetic field are externally excited in a straight cylindrical plasma column and the response of the plasma ions is measured by coherent detection of laser induced fluorescence. A sketch of the experimental set-up is shown in figure 2. We define the linear response function as the phase-coherent response of the ion distribution function to a harmonic externally applied signal. Both the coherent and the incoherent response of the ions to the applied waves are measured. The response is measured as a function of position and velocity along the magnetic field and of wave frequency. Figure 3 shows recent high-resolution data on the linear response that is plotted as a function of position and particle velocity for 50 kHz excitation. Both an in-phase and a quadrature component are measured, but only the real part is shown in the figure. If we plot a section of the data we observe that even at several ion acoustic wavelengths from the antenna, the plasma response cannot be described by the ion acoustic wave alone (figure 4). This fact, in itself, has been known for many years and has been attributed to the continuum. The new aspect is that all phase velocities are not equivalent from the point of view of the plasma response. First, there is the ion acoustic velocity, and second we observe other phase velocities. We pose the question: how can phase mixing occur in a system that does not have a continuum?

A Fourier transform of the data produces a response function which, for a given frequency, is resolved in both particle velocity and perturbation phase velocity. Thus we experimentally determine the function

$$f_1\left(\omega, \frac{\omega}{k_{\parallel}}, v_{\parallel}\right) = \int_0^{\infty} v_{\perp} dv_{\perp} \sum_{n=0}^{\infty} \exp(in\varphi) A(k_{\parallel}, \omega, n, v_{\parallel}, v_{\perp}, k_{\perp}) \quad (5)$$

where we note that k_{\perp} is observed to be as small as the geometry of the plasma will allow. A contour plot of the magnitude of the complex function f_1 in the particle velocity-phase velocity plane shows the different mode velocities as well as the lines that correspond to wave-particle resonance. In this representation of the data, collective modes are narrow bands parallel to the particle-velocity axis. The width of the bands is related to the spatial damping of the collective modes. Because of mode damping, a singular value decomposition (SVD) is a better way to identify separate modes than the Fourier transform. The SVD performs a projection of the data onto orthogonal modes of spatial and velocity structure according to the form: $f_1 = \sum_m \Lambda_m G_m(z) H_m(v_{\parallel})$. Because the true eigenfunctions are not guaranteed

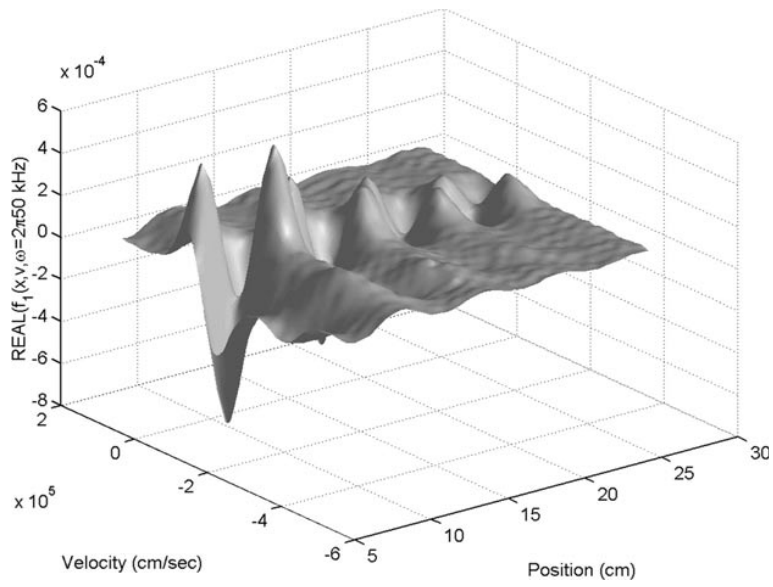


Figure 3. Real part of the plasma response to a sinusoidal input at 50 kHz as a function of particle velocity and distance from the antenna.

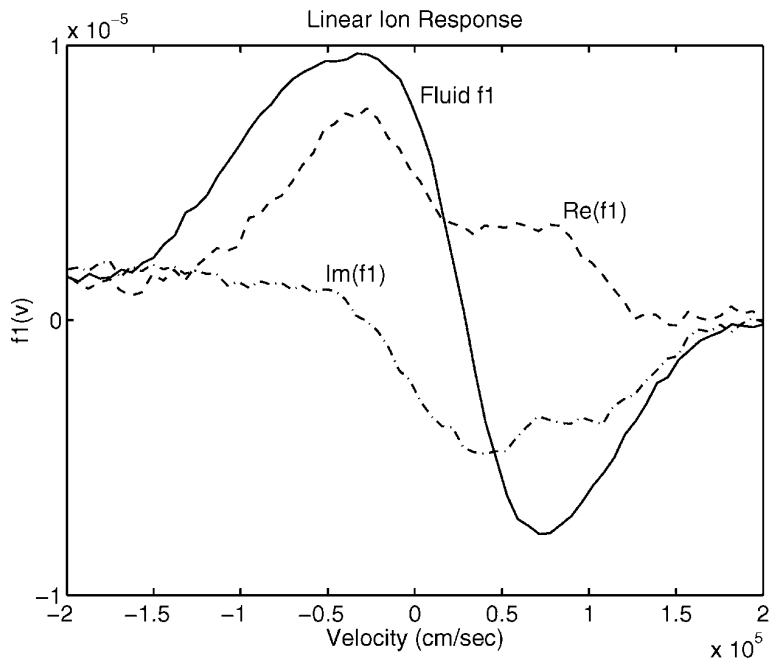


Figure 4. Plot of the plasma response as a function of particle velocity only. If the plasma response were controlled by fluid theory then the response would be as given by the full curve with a negligible imaginary part.

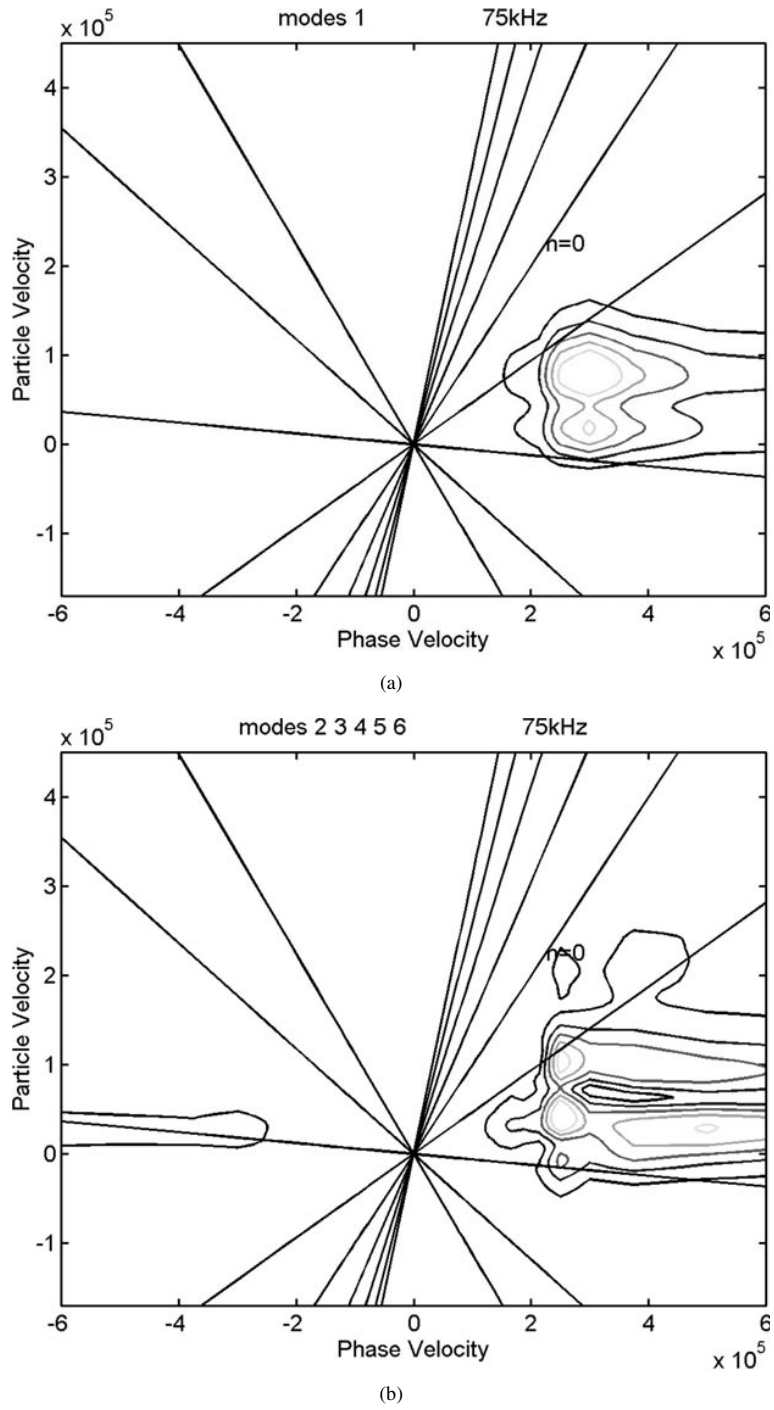


Figure 5. Plasma response magnitude as a function of particle velocity and collective mode phase-velocity. The condition for wave-particle resonance is specified by straight lines for each n . (a) The electrostatic ion cyclotron wave. (b) Modes not given by fluid theory.

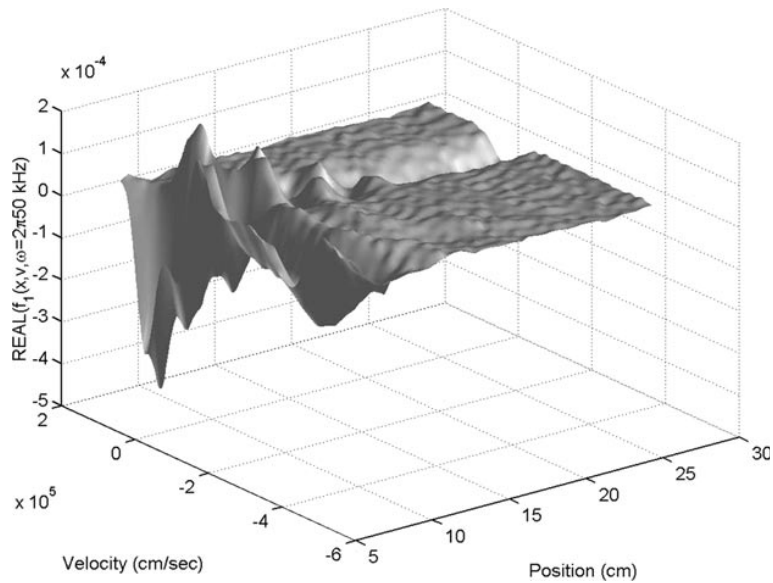


Figure 6. Response function for an excitation amplitude above the threshold for chaotic particle motion.

to be orthogonal functions of velocity (they are orthogonal to the adjoint eigenfunctions) this SVD approach may also introduce errors, but we observe that when applied to calculated eigenfunctions of equation (4) the procedure works surprisingly well. Figure 5(a) shows the structure of the observed electrostatic ion cyclotron wave (oblique ion acoustic wave in a magnetic field). This mode corresponds closely to what one expects from the fluid theory. Figure 5(b) however, shows other modes in the data. We observe that the phase velocity does correspond to the phase velocity of a discrete kinetic mode predicted by equation (4). However, the damping is much less than we can currently account for. Even if the SVD is giving us a false indication of the damping decrement, the damping is also bounded by the magnitude of the signal and the distance from the antenna. Another possible source of error is nonlinearity, so the experiments were performed at a low perturbation intensity of $<5\%$.

Waves with a small perpendicular wavenumber will have narrow cyclotron harmonic resonances and thus may have resonant particles but weak damping. Narrow resonances limit the diffusion of (the small number of) chaotic orbits at low amplitude. Except for the fundamental ($n = 0$) resonance, the linear theory will tend to hold because the bounce time can be made to be long compared with the collision time. We are in a regime where linear theory should not be limited by timescales. The use of increased amplitude produces a dramatic effect when the threshold for wide ranging chaotic motion is exceeded ($\sim 10\%$ density perturbation). In addition to particle heating, the wave amplitude at the points of measurement actually decreases relative to the lower excitation amplitude due to the nonlinear absorption process (figure 6). In addition, the SVD mode spectrum of the data is strongly modified for the heated distributions. We note that for very low collision frequency it is possible to excite a continuum of nonlinear waves even at low amplitude [10]. Measurements of the coherent three-wave interaction resolved in particle velocity are also being studied to see how the plasma degrees of freedom determine the nature of wave-wave interactions [11].

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