

6. Analysis of multispectral images

6.1. Spectral transforms

(Reading: Schowengerdt, 1997, pp. 179-227)

- each pixel in a multispectral image of k channels can be represented by a spectral vector \mathbf{DN} (based on the digital number for each spectral channel) in a k -dimensional spectral space; a linear spectral transform \mathbf{W} results in a transformed pixel or scene based on the matrix product

$$\mathbf{DN} \otimes \mathbf{W} \bullet \mathbf{DN}$$

$$\mathbf{DN} \otimes \begin{bmatrix} w_{11} & \dots & w_{1k} \\ \dots & \dots & \dots \\ w_{k1} & & w_{kk} \end{bmatrix} \begin{bmatrix} DN_1 \\ \dots \\ DN_k \end{bmatrix} \quad (6.1)$$

- one of the simplest, fundamental spectral transforms is the computation of multispectral ratios $R_{mn}(x,y)$ at a point x,y in an image between spectral bands m and n :

$$R_{mn}(x,y) = \frac{DN_m(x,y)}{DN_n(x,y)} \quad (6.2)$$

- digitization of a visible-range multispectral scene results in digital numbers for the different bands b that are approximately equal to the product of the the surface reflectance $\rho_b(x,y)$ and the cosine of the solar incidence angle $\theta(x,y)$, with gain a and offset b as defined in equation 1.6:

$$DN_b(x,y) \approx a_b \rho_b(x,y) \cos[\theta(x,y)] + b_b \quad (6.3)$$

- channel ratioing represents a useful transform because it eliminates the incident topographic irradiance factor from equation 6.3, such that for a scene with b_b equal for all channels, the multispectral ratio is given by

$$R_{mn}(x,y) \approx \frac{a_m \rho_m(x,y)}{a_n \rho_n(x,y)} = k_{mn} \frac{\rho_m(x,y)}{\rho_n(x,y)} \quad (6.4)$$

- thus the channel ratio R_{mn} and a variant, the modulation ratio M_{mn} defined as

$$M_{mn} = \frac{R_{mn} - 1}{R_{mn} + 1} \quad (6.5)$$

are more characteristic of the true surface properties rather than differences in illumination across a scene, effectively removing even shadowing effects and other modifications of the surface (such as wetting etc.)

- a widely employed multispectral ratio is the so-called ratio vegetation index (RVI) and its variant, the normalized difference vegetation index (NDVI), defined as the reflectance ratio in the near-infrared and the red band (ideally for a given spectral range or sensor band, such as channels 1 and 2 of the AVHRR instrument):

$$RVI = \frac{\rho_{NIR}}{\rho_{red}} \quad (6.6)$$

$$NDVI = \frac{\rho_{NIR} - \rho_{red}}{\rho_{NIR} + \rho_{red}}$$

- vegetation indices discriminate between unvegetated and vegetated surfaces through the characteristic increase in reflectance for plant biomass in the near-IR range

- in many applications (such as satellite remote sensing based on, e.g., Landsat TM or SPOT data sets), a significant degree of correlation between individual channels in multispectral data sets can be observed; derivation of the principal components (PCs) of such a data set often allows for a significant reduction in the number of composite images required to adequately describe and discriminate data and furthermore aids in image processing such as the reduction of terrain shading effects etc.

- the principal component transform is a linear transform as shown in equation 6.1, such that

$$\mathbf{PC} = \mathbf{W}_{PC} \bullet \mathbf{DN} \quad (6.7)$$

where \mathbf{W}_{PC} is an image specific matrix; \mathbf{W}_{PC} is determined such that it diagonalizes the covariance matrix \mathbf{C} (thereby reducing the correlation to that between a set of orthogonal (eigen)vectors \mathbf{e}_k); thus, the covariance matrix is transformed by \mathbf{W}_{PC} such that

$$\mathbf{C}_{PC} = \mathbf{W}_{PC} \mathbf{C} \mathbf{W}_{PC}^T \quad (6.8)$$

- the covariance matrix is given by

$$\mathbf{C} = \frac{1}{K-1} \sum_{j=1}^K (DN_j - \langle DN \rangle)(DN_j - \langle DN \rangle)^T \quad (6.9)$$

- the diagonalized covariance matrix \mathbf{C}_{PC} is constituted of the eigenvalues λ_k of \mathbf{C} :

$$\mathbf{C}_{PC} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & \lambda_K \end{bmatrix} \quad (6.10)$$

which can be determined as the K roots of the characteristic equation

$$\mathbf{C} - \lambda \mathbf{I} = 0 \quad (6.11)$$

where \mathbf{I} is the (diagonal) identity matrix

- the PCs thus correspond to the (orthogonal) coordinate axes in a rotated multispectral vector space, with each axis defined by one of the K eigenvectors \mathbf{e}_k which follow from the characteristic equation for each eigenvalue

$$(\mathbf{C} - \lambda_k \mathbf{I})\mathbf{e}_k = 0 \quad (6.12)$$

- these eigenvectors form the rows of the transformation matrix \mathbf{W}_{PC} :

$$\mathbf{W}_{PC} = \begin{bmatrix} e_1^T \\ \dots \\ e_K^T \end{bmatrix} = \begin{bmatrix} e_{11} & \dots & e_{1K} \\ \dots & \dots & \dots \\ e_{K1} & \dots & e_{KK} \end{bmatrix} \quad (6.13)$$

- the matrix product of 6.7 can thus be interpreted as a coordinate transformation, such that the data \mathbf{DN} are projected onto new coordinates \mathbf{PC} that minimize the degree of correlation between individual channels; this is illustrated in Fig. 6.1 that shows an exemplary scatterplot for two bands of highly correlated data (with considerable overlap in the individual channels 1 and 2) and the same data projected onto new axes that maximize the variance along these axes

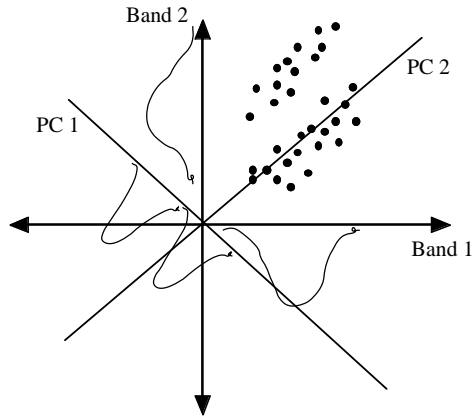


Fig. 6.1: Coordinate transformation based on Principal components analysis, such that projection of data clusters onto PC_1 results in better separability of data clusters.