Notes on Electrical Properties, further to the class ppt

The electrical conductivity is defined via $J = \sigma E$. This is related to the probably more familiar Ohm's Law, $I = V/R$, where I [amps, A] is the current resulting from applied potential difference V [voltage drop, V] across a sample, and R is the electrical resistance [ohms, Ω].

To get to Ohm's Law, use $J = I/A$ (current density = current per unit area), $E = V/L$ and, finally, $R = \rho L / A$ where the resistivity ρ is the inverse of conductivity, $\rho = 1/\sigma$. Resistivity is the intrinsic property, resistance depends on sample dimensions.

The mathematical similarity between electrical conduction and fluid permeability is more obvious when we recall that electrical fields represent gradients of electric potential, $E = -\nabla\phi$. Then we have: $J = -\sigma\nabla\phi$ for electrical conductivity, and $q = -(\kappa/\eta)\nabla p$ for fluid flow.

Electrical Conductivity Temperature Dependence

The electrical conductivity of metals decreases with increasing temperature, but increases for semi-conductors. Below is a simplified sketch of why. Phonons again play a role! (A fuller explanation involves discussion of energy bands in solids, eg. Kittel, or www.)

Metals:
Conduction pictured as the motion of free electrons in an "electron gas". This is a similar picture to the "phonon gas" of heat transport in crystal lattices. In both cases it is phonons that are the main scattering agent. Once again, the distance between scattering collisions is the mean free path. Or we can think of the collision time, being the time between collisions.

In metals, electrons are largely responsible for both heat and charge transport, the thermal (k) and electrical (σ) conductivities are proportional, as related by the Weidemann Franz law: $k / \sigma = LT$, where T is temperatures [Kelvin] and $L = 2.45\times10^{-8}$ W Ω K^{-2}. Actually, this is an empirical relation rather than a 'law'.

With increasing temperature the number density of phonons increases linearly, and in proportion so does the scattering of electrons, so that the electrical conductivity decreases as $\sigma \sim 1/T$ (for T in Kelvin). As $T \rightarrow 0$, there are fewer phonons activated, but scattering by impurities means that the *resistivity* doesn't go to 0 and the conductivity remains finite.

Semiconductors:
The conduction picture in semiconductors is more complicated. Rather than a 'sea of free electrons', we can think of the electrons as being more tightly bound to the atoms. They are said to reside in 'valence bands', which are separated by an 'energy gap' from the 'conduction band' in which the electrons *would* be free to move.

Statistically, individual electrons can 'borrow' from the available thermal energy to make the jump to the conduction band more readily as the average thermal energy increases – ie. as the temperature increases. In particular, this kind of jumping an energy gap Eg is mathematically characterized by an exponential factor which is similar in form to other thermally-activated threshold processes. (See class ppt and Arrhenius plot).