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A re-evaluation of the Webb correction using density-weighted averages

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Abstract

Results from a re-evaluation of the flux correction suggested by Webb et al. (*Q. J. R. Meteorol. Soc.*, 106, 85–100, 1980) are presented and discussed. This re-evaluation is based on the equation of continuity as well as the budget equations for dry air, water vapour and atmospheric trace species, where a density-weighted averaging procedure introduced by Hesselberg (*Beitr. Phys. fr. Atmos.*, 12, 141–160, 1926) is used. This averaging procedure seems to be more appropriate than that of Reynolds, especially in the case of atmospheric trace species. The consequences of this flux correction as regards the exchange of atmospheric trace gases between the atmosphere and the ground (vegetation, soil and water) are pointed out.

1. Introduction

Webb and Pearman (1977), Bakan (1978), Smith and Jones (1979), Webb et al. (1980), Bernhardt and Piazena (1988) and Foken (1989), among others, suggested that in the case of trace constituents (such as CO₂), the average concentrations of which are very large compared with the concentration fluctuations, the measurements of eddy fluxes in the atmospheric boundary layer have to be corrected for air density fluctuations. Such density fluctuations occur, for instance, because of water vapour fluctuations. Most of the corrections derived by these workers are based on the assumption that the mean vertical flux of dry air constituent should be zero, i.e.

$$\overline{\rho_a w} = 0 \quad (1)$$

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which, with the left-hand side written as $\overline{\rho_a w} + \rho_a' w'$, leads to

$$\overline{w} = -\frac{\overline{\rho_a' w'}}{\rho_a} \quad (2)$$

Here, ρ_a is the density of dry air, and w is the vertical wind component. The overbar denotes the simple average and a prime the deviation from that.

Following Webb et al. (1980), the density fluctuation ρ_a' can be expressed by

$$\rho_a' = -\mu \rho_v' - \overline{\rho_a} (1 + \mu \sigma_v) \frac{T'}{\overline{T}} \quad (3)$$

where $\mu = m_a/m_v$ is the ratio of the molecular weights of dry air and water vapour, ρ_v is the partial density of water vapour, $\sigma_v = \overline{\rho_v}/\overline{\rho_a}$, and T is the air temperature. Using this expression for the density fluctuation, Eq. (2) becomes

$$\overline{w} = \mu \frac{\overline{\rho_v' w'}}{\overline{\rho_a}} + (1 + \mu \sigma_v) \frac{\overline{w' T'}}{\overline{T}} \quad (4)$$

As suggested by Webb et al. (1980), the total flux of an entity with the partial density ρ_c can be expressed by

$$F = \overline{\rho_c w} = \overline{\rho_c} \overline{w} + \overline{\rho_c' w'} \quad (5)$$

so that after introducing Eq. (4) into Eq. (5) the flux equation is given by

$$F = \overline{\rho_c' w'} + \sigma_c \mu \overline{\rho_v' w'} + \overline{\rho_c} (1 + \mu \sigma_v) \frac{\overline{w' T'}}{\overline{T}} \quad (6)$$

with $\sigma_c = \overline{\rho_c}/\overline{\rho_a}$. This flux correction is often called the Webb correction. It was used, for example, by Webb et al. (1980) to correct measured CO_2 fluxes.

However, as shown in Appendix B, the mean vertical wind component can also be expressed by

$$\overline{w} = -\frac{1}{\overline{\rho} (1 - \overline{\rho_v}/\overline{\rho})} [(1 - \overline{q}) \overline{\rho' w'} - \overline{\rho} \overline{w' q'} - \overline{\rho' w' q'}] \quad (7)$$

Here, $q = \rho_v/\rho$ is the specific humidity. If the third-order moment is neglected and the total density fluctuation is approximated by $\rho' \approx -T' \overline{\rho}/\overline{T}$ (this assumes that pressure fluctuations are negligible; see, e.g. Lumley and Panofsky (1964)), the mean vertical wind component becomes

$$\overline{w} \approx \frac{1}{1 - \overline{\rho_v}/\overline{\rho}} \left[(1 - \overline{q}) \frac{\overline{w' T'}}{\overline{T}} + \overline{w' q'} \right] \quad (8)$$

Usually, $\overline{q} \ll 1$ and $\overline{\rho_v}/\overline{\rho} \ll 1$ so that the mean vertical wind component can be approximated by

$$\overline{w} \approx \frac{\overline{w' T'}}{\overline{T}} + \overline{w' q'} \quad (9)$$

As shown by Webb et al. (1980), the quantities $\overline{w' T'}$ and $\overline{w' q'}$ may be expressed in terms of the sensible heat flux, $H = c_p \overline{\rho} \overline{w' T'}$, and the Bowen ratio, $\beta = H/E$, where

$E = L_v \overline{\rho w' q'}$ is the latent heat, and L_v is the latent heat of vaporization, to discuss the relative importance of both flux correction terms, i.e.

$$\overline{w} \approx \frac{H}{c_p \overline{\rho}} \left(\frac{1}{T} + \frac{c_p}{\beta L_v} \right) \quad (10)$$

Eq. (10) shows that the flux correction depends largely on the sensible heat flux, whereas the effects owing to water vapour fluctuations are negligible except when the Bowen ratio is small.

Here, the Webb correction is re-evaluated, i.e. proceeding from Eq. (1), equations both for the mean vertical wind component and for the eddy flux of a trace constituent are derived; in contrast to Webb et al. (1980), the equation of continuity as well as the budget equations for dry air, water vapour and the trace species are taken into consideration (Bernhardt and Piazena, 1988). Furthermore, it is advantageous to use the density-weighted averaging procedure introduced by Hesselberg (1926), which appears to be more appropriate than that of Reynolds, especially in the case of atmospheric trace constituents.

2. Theoretical background

2.1. The density-weighted vertical wind component in the moist atmospheric surface layer

The density ρ of moist air is given by

$$\rho = \rho_a + \rho_v \quad (11)$$

Eq. (1) can be expanded to yield

$$\overline{\rho_a w} = \overline{(\rho_a + \rho_v) w} - \overline{\rho_v w} = 0 \quad (12)$$

According to Hesselberg's (1926) averaging procedure (see also Van Mieghem, 1949, 1973, 1974; Montgomery, 1954; Herbert, 1975), an extensive specific quantity $\varphi = \Phi/M$ (where M is mass) can be split up into two parts

$$\varphi = \langle \varphi \rangle + \varphi'' \quad (13)$$

where $\langle \varphi \rangle$ is defined as

$$\langle \varphi \rangle = \frac{\overline{\rho \varphi}}{\overline{\rho}} \quad (14)$$

and where the mean value of φ'' is zero, i.e.

$$\langle \varphi'' \rangle = 0 \quad (15)$$

The brackets $\langle \rangle$ denote the density-weighted average and the double prime marks the departure from that. It should be noted that $\langle \varphi' \rangle = -\varphi'' \neq 0$. The advantage of the density-weighted average is, among others, that, after averaging, the equation of continuity and of state have the same forms as before, i.e. (see Hesselberg, 1926)

equation of continuity:

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \langle \mathbf{v} \rangle) = 0 \quad (16)$$

equation of state:

$$\bar{p} = \bar{\rho} R \langle T_v \rangle \quad (17)$$

The quantity $\langle \mathbf{v} \rangle$ is the density-weighted wind vector, and $\langle T_v \rangle = \langle T \rangle (1 + 0.61 \langle q \rangle + 0.61 \langle q'' T'' \rangle / \langle T \rangle)$ is the density-weighted virtual temperature, where the term $0.61 \langle q'' T'' \rangle / \langle T \rangle$ is negligible, and, hence, $\langle T_v \rangle \approx \langle T \rangle (1 + 0.61 \langle q \rangle)$. A further advantage of a density-weighted average is that the mean value of the kinetic energy can be separated exactly into the kinetic energy of the mean motion and the mean value of the kinetic energy of the eddying motion expressed by (see Van Mieghem, 1949, 1974)

$$\frac{1}{2} \overline{\rho v^2} = \frac{1}{2} \bar{\rho} \langle \mathbf{v} \rangle^2 + \frac{1}{2} \overline{\rho (\mathbf{v}'')^2} \quad (18)$$

Eq. (12) can be rearranged to give

$$\bar{\rho}_a \bar{w} = \bar{\rho} \langle w \rangle - \overline{\rho q w} = 0 \quad (19)$$

where the second term on the right-hand side of Eq. (19) is given by

$$\overline{\rho q w} = \bar{\rho} \langle q \rangle \langle w \rangle + \overline{\rho q w''} \quad (20)$$

Here, $\overline{\rho q w''}$ is the vertical eddy flux component of water vapour. Combining Eqs. (19) and (20) provides

$$\bar{\rho}_a \bar{w} = \bar{\rho} \langle w \rangle (1 - \langle q \rangle) - \overline{\rho q w''} = 0 \quad (21)$$

so that the density-weighted vertical wind component can be expressed by (see also Appendix C)

$$\langle w \rangle = \frac{\overline{\rho q w''}}{\bar{\rho} (1 - \langle q \rangle)} = \bar{w} + \frac{\overline{\rho' w'}}{\bar{\rho}} \quad (22)$$

Hence, the water vapour flux induces a mean vertical velocity. Usually, $\langle q \rangle \ll 1$. Therefore, $\langle w \rangle$ can be approximated by

$$\langle w \rangle = \frac{1}{\bar{\rho}} \overline{\rho q w''} \quad (23)$$

2.2. The budget equations for dry air, water vapour and trace constituents

The budget equations for dry air, water vapour and trace constituents with Hesselberg averaging are given by

$$\frac{\partial \bar{\rho}_k}{\partial t} + \nabla \cdot (\bar{\rho}_k \langle \mathbf{v} \rangle) + \nabla \cdot (\overline{\rho_k \mathbf{v}''} + \overline{\mathbf{II}_k}) = \bar{I}_k \quad (24)$$

Here, index k refers to dry air ($k = a$), water vapour ($k = v$) and trace constituents

($k = c$), and \bar{I}_k represents the corresponding sources and sinks, where \bar{I}_a is always equal to zero. Within the framework of this study, no processes of phase transition and chemical transformations are considered. Therefore, \bar{I}_v and \bar{I}_c are equal to zero, too. The flux quantity \bar{II}_k may represent molecular, phoretic and sedimentation effects. Because in the turbulent region of the atmospheric surface layer the flux quantity \bar{II}_k is small in comparison with the turbulent flux $\overline{\rho_k v''}$, \bar{II}_k is neglected in the following.

The air density is given by

$$\rho = \rho_a + \rho_v + \rho_c \quad (25)$$

where, however, $\rho_c \ll \rho_v$ and, hence, $c = \rho_c/\rho \ll q$, so that ρ can be taken from Eq. (11). It is obvious that Eq. (24) fulfils the equation of continuity (16), because the total divergence of the eddy fluxes of dry air, water vapour and trace constituents is always equal to zero, i.e.

$$\nabla \cdot \left(\sum_k \overline{\rho_k v''} \right) = \nabla \cdot (\overline{\rho v''}) = \nabla \cdot (\overline{\rho(v'')}) = 0 \quad (26)$$

Assuming that steady-state conditions exist during the measuring interval, Eq. (24) leads to

$$\nabla \cdot (\overline{\rho_a v}) + \nabla \cdot (\overline{\rho_a v''}) = 0 \quad (27)$$

$$\nabla \cdot (\overline{\rho(q)v}) + \nabla \cdot (\overline{\rho q v''}) = 0 \quad (28)$$

$$\nabla \cdot (\overline{\rho(c)v}) + \nabla \cdot (\overline{\rho c v''}) = 0 \quad (29)$$

As $\overline{\rho_a v} = \overline{\rho_a v} + \overline{\rho_a v''}$, Eq. (27) can be rearranged to give

$$\nabla_H \cdot (\overline{\rho_a v_H}) + \frac{\partial}{\partial z} (\overline{\rho_a w}) = 0 \quad (30)$$

The index H denotes the horizontal part of the vector operation. Because of Eq. (1), both terms $\partial(\overline{\rho_a w})/\partial z$ and $\nabla_H \cdot (\overline{\rho_a v_H})$ are equal to zero, i.e. there exists no sources and sinks for the flux of dry air. Hence, Eq. (1) is compatible with horizontal homogeneity.

Assuming also horizontal homogeneity, Eqs. (28) and (29) reduce to

$$\frac{\partial}{\partial z} (\overline{\rho(q)w}) + \overline{\rho q w''} = 0 \quad (31)$$

and

$$\frac{\partial}{\partial z} (\overline{\rho(c)w}) + \overline{\rho c w''} = 0 \quad (32)$$

where $\overline{\rho c w''}$ is the vertical eddy flux of a trace species. Taking Eq. (22) into account,

$$\frac{\partial}{\partial z} \left(\frac{1}{1 - \langle q \rangle} \overline{\rho q w''} \right) = 0 \quad (33)$$

and, hence,

$$\frac{1}{1 - \langle q \rangle} \overline{\rho q w''} \approx \overline{\rho q w''} = \text{constant} \quad (34)$$

as well as

$$\frac{\partial}{\partial z} \left(\frac{\langle c \rangle}{1 - \langle q \rangle} \overline{\rho q w''} + \overline{\rho c w''} \right) = 0 \quad (35)$$

and, hence,

$$\begin{aligned} \frac{\partial}{\partial z} \overline{\rho c w''} &= - \frac{1}{1 - \langle q \rangle} \overline{\rho q w''} \frac{\partial \langle c \rangle}{\partial z} \\ &\quad - \langle c \rangle \frac{\partial}{\partial z} \left(\frac{1}{1 - \langle q \rangle} \overline{\rho q w''} \right) \\ &= 0 \quad (\text{see Eq. (33)}) \end{aligned} \quad (36)$$

Therefore, as $\langle q \rangle \ll 1$,

$$\frac{\partial}{\partial z} (\overline{\rho c w''}) \approx - \overline{\rho q w''} \frac{\partial \langle c \rangle}{\partial z} \quad (37)$$

It is obvious that the right-hand side of either Eq. (36) or (37) does not vanish, i.e. the water vapour flux along with the mean concentration are responsible for the fact that the eddy flux of a trace species depends on height.

Integrating Eq. (36) for the layer $0 \leq z \leq z_R$ (where z_R is the reference height) provides

$$F_R = \overline{\rho c w''} \Big|_{z_R} = F_S - \frac{\langle c_R \rangle - \langle c_S \rangle}{1 - \langle q \rangle} \overline{\rho q w''} \quad (38)$$

Here, F_S is the flux of a trace constituent at the surface (measured, for instance, by enclosure techniques), $\langle c_S \rangle$ is the corresponding mean concentration, F_R is the eddy flux of the trace constituent at z_R , and $\langle c_R \rangle$ is the mean concentration at the same height. After introducing (22) into Eq. (38),

$$F_R = F_S - (\langle c_R \rangle - \langle c_S \rangle) \bar{\rho} \langle w \rangle = F_S - (\bar{\rho}_{c,R} - \bar{\rho}_{c,S}) \langle w \rangle \quad (39)$$

This equation illustrates that the relative importance of the flux correction is dependent both on the mean vertical wind component and on the density difference $\Delta \bar{\rho}_c = \bar{\rho}_{c,R} - \bar{\rho}_{c,S}$.

3. Results and conclusions

Taking Eq. (22) and the Bowen ratio $\beta = H/E$ into account, the density-weighted vertical wind component can be expressed

$$|\langle w \rangle| = \frac{|H|}{\bar{\rho}(1 - \langle q \rangle)|\beta|L_v} \quad (40)$$

and as $\langle q \rangle \ll 1$,

$$|\langle w \rangle| \approx \frac{|H|}{\bar{\rho}|\beta|L_v} \quad (41)$$

Here, $H = c_{p,o} \overline{\rho \Theta w''}$ is the eddy flux of sensible heat, $E = L_v \overline{\rho q w''}$ is the eddy flux of latent heat (according to Hesselberg's averaging procedure; see Herbert, 1975), $c_{p,o}$ is the specific heat at constant pressure for dry air, Θ is the potential temperature, and $L_v \approx 2.5 \times 10^6 \text{ J kg}^{-1}$ is the heat of vaporization. Fig. 1 shows results which were derived for arbitrary values of $\bar{\rho} = 1.2 \text{ kg m}^{-3}$, $|\beta| = 0.1, 1.0$ and 10.0 , and $|H| = 100.0, 200.0, 300.0$ and 400.0 W m^{-2} , where $\langle q \rangle$ was neglected. As illustrated in this figure, in most cases the density-weighted vertical wind component (see Eq. (22)) induced by water vapour fluctuations is negligible except for small values of $|\beta|$ and large values of $|H|$ which correspond to very large latent heat fluxes.

Hence, in most cases the term $\Delta \bar{\rho}_c \langle w \rangle$ in Eq. (39) can be neglected because $\Delta \bar{\rho}_c$ and F_S cannot be considered as entirely independent of each other.

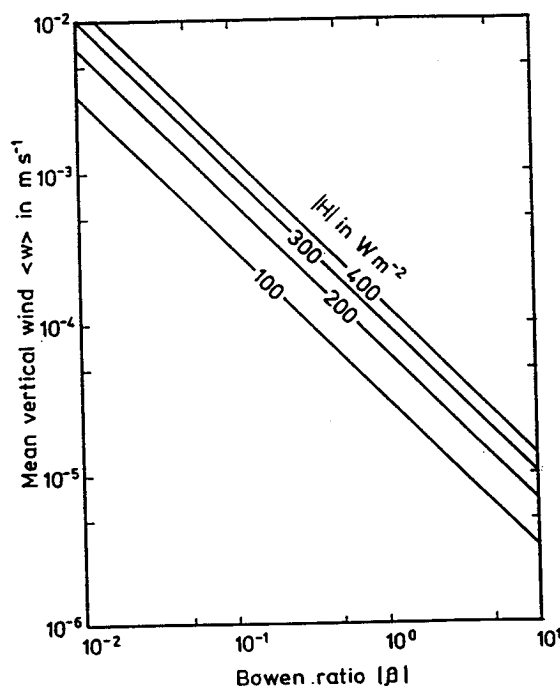


Fig. 1. The mean vertical wind component, $\langle w \rangle$, as a function of the eddy flux of sensible heat, $|H|$, and the Bowen ratio $|\beta|$.

Appendix A

Following Hesselberg (1926) and Herbert (1975), the mean quantities are given by: Reynolds averaging:

$$\bar{\varphi} = \bar{\varphi}(r) = \frac{1}{G} \int_G \varphi(r, r') dG' \quad (\text{A1})$$

Hesselberg averaging:

$$\langle \varphi \rangle = \langle \varphi(r) \rangle = \frac{\int_G \rho(r, r') \varphi(r, r') dG'}{\int_G \rho(r, r') dG'} = \frac{\bar{\rho\varphi}}{\bar{\rho}} \quad (\text{A2})$$

Here, r is the four-dimensional 'vector' of space and time in the original coordinate system, r' is that of the averaging domain G , where its origin, $r' = 0$, is assumed to be r , and $dG' = d^3r' dt'$. The averaging domain G is given by $G = \int_G dG'$. The quantities $\bar{\varphi}$ and $\langle \varphi \rangle$ represent also the mean values of φ at the location r in the averaging domain. It is obvious that $\bar{\varphi}' = 0$, and $\langle \varphi'' \rangle = 0$.

The following arithmetic rules can be used (see Van Mieghem, 1949; Lumley and Panofsky, 1964; Herbert, 1975):

$$\overline{\varphi + \chi} = \bar{\varphi} + \bar{\chi}, \quad \langle \varphi + \chi \rangle = \langle \varphi \rangle + \langle \chi \rangle \quad (\text{A3})$$

$$\overline{\alpha\varphi} = \alpha\bar{\varphi}, \quad \langle \alpha\varphi \rangle = \alpha\langle \varphi \rangle, \quad \text{if } \alpha = \text{constant} \quad (\text{A4})$$

$$\bar{\bar{\varphi}} = \bar{\varphi}, \quad \langle \langle \varphi \rangle \rangle = \langle \varphi \rangle \quad (\text{A5})$$

$$\overline{\langle \varphi \rangle} = \bar{\varphi}, \quad \langle \overline{\varphi} \rangle = \langle \varphi \rangle \quad (\text{A6})$$

$$\langle \varphi' \rangle = -\overline{\varphi''} \neq 0 \quad (\text{A7})$$

$$\overline{\frac{\partial \varphi}{\partial t}} = \frac{\partial \bar{\varphi}}{\partial t}, \quad \overline{\nabla \varphi} = \nabla \bar{\varphi}, \quad \overline{\nabla \cdot \varphi} = \nabla \cdot \bar{\varphi} \quad (\text{A8})$$

$$\overline{\varphi\chi} = \bar{\varphi}\langle \chi \rangle + \overline{\varphi\chi''} = \bar{\varphi}\bar{\chi} + \overline{\varphi\chi'} \quad (\text{A9})$$

$$\overline{\rho\varphi\chi} = \bar{\rho}\langle \varphi \rangle \langle \chi \rangle + \overline{\rho\varphi''\chi''} \quad (\text{A10})$$

Appendix B

The fluctuation of the air density is given by

$$\rho' = \rho - \bar{\rho} = \rho_a + \rho_v - \overline{\rho_a + \rho_v} = \rho_a' + \rho_v' \quad (\text{B1})$$

As the specific humidity is defined by $q = \rho_v/\rho$, the fluctuation of the water vapour density can be expressed by

$$\rho'_v = \rho_v - \bar{\rho}_v = \rho'q + \bar{\rho}q' + \rho'q' - \bar{\rho}'q' \quad (\text{B2})$$

Thus, the fluctuation of the dry air density becomes

$$\rho'_a = \rho' - \rho'_v = \rho' - \rho'q - \bar{\rho}q' - \rho'q' + \bar{\rho}'q' \quad (\text{B3})$$

Multiplying Eq. (B3) with w' and averaging yields the exact relation

$$\overline{\rho'_a w'} = \overline{\rho' w'}(1 - \bar{q}) - \bar{\rho} \overline{w' q'} - \overline{\rho' w' q'} \quad (\text{B4})$$

Taking Eq. (2) and $\bar{\rho}_a = \bar{\rho}(1 - \bar{\rho}_v/\bar{\rho})$ into account, the mean vertical wind component becomes

$$\bar{w} = -\frac{1}{\bar{\rho}(1 - \bar{\rho}_v/\bar{\rho})} [(1 - \bar{q})\overline{\rho' w'} - \bar{\rho} \overline{w' q'} - \overline{\rho' w' q'}] \quad (\text{B5})$$

Appendix C

To derive a relationship between the Reynolds mean wind vector and the density-weighted wind vector, it is advantageous to consider the equation of continuity formulated for both cases, i.e.

Reynolds averaging:

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{\mathbf{v}} + \overline{\rho' \mathbf{v}'}) = 0 \quad (\text{C1})$$

Hesselberg averaging:

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \langle \mathbf{v} \rangle) = 0 \quad (\text{C2})$$

Combining these equations yields

$$\nabla \cdot (\bar{\rho} \bar{\mathbf{v}} + \overline{\rho' \mathbf{v}'} - \bar{\rho} \langle \mathbf{v} \rangle) = 0 \quad (\text{C3})$$

which leads to a constant spatial vector, i.e.

$$\bar{\rho} \bar{\mathbf{v}} + \overline{\rho' \mathbf{v}'} - \bar{\rho} \langle \mathbf{v} \rangle = \text{constant} \quad (\text{C4})$$

As the mean wind vector and also the fluctuating part vanish at the Earth's surface, the magnitude of the constant vector is always equal to zero. Thus, one obtains

$$\langle \mathbf{v} \rangle = \bar{\mathbf{v}} + \frac{\overline{\rho' \mathbf{v}'}}{\bar{\rho}} \quad (\text{C5})$$

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