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Discussion

A re-evaluation of the Webb correction using density-weighted averages — Reply

Gerhard Kramm

Fraunhofer-Institut für Atmosphärische Umweltforschung, Kreuzeckbahnstraße 19, D-82467 Garmisch-Partenkirchen, Germany

The author gratefully accepts Herbert's (1995) comment. The use of the equation of continuity also in the Appendix C of Kramm et al. (1995; referred to as KDL hereafter) without any consideration of Eq. (A9) of KDL to relate Hesselberg's (1926) vertical wind component, $\langle w \rangle$, to that of Reynolds, $\bar{w}$, by (same notation as in KDL)

$$\langle w \rangle = \bar{w} + \overline{\rho' w'}/\bar{\rho} \tag{1}$$

(applied in Eq. (22) of KDL) seems to be somewhat overstrained. Herbert's direct derivation of this relation on the basis of (see Eq. (A9) of KDL)

$$\overline{\varphi\chi} = \bar{\varphi}\langle\chi\rangle + \overline{\varphi\chi''} = \bar{\varphi}\bar{\chi} + \overline{\varphi\chi'} \tag{2}$$

has to be preferred because with $\varphi = \rho, \chi = v$, and $\langle v'' \rangle = 0$ one obtains immediately, i.e. independently of the arguments given in the Appendix C of KDL, the desired relation (see Eq. (C5) of KDL)

$$\langle v \rangle = \bar{v} + \overline{\rho v'}/\bar{\rho} \tag{3}$$

In particular, Eq. (2) was used by KDL to derive their central Eq. (16), i.e. the averaged equation of continuity according to Hesselberg (1926). The prominent relevance of this equation of continuity is in full agreement with Herbert's discussion because it is a result of the general validity of Eq. (3) owing to averaging rules.

References

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F. Herbert

Institut für Meteorologie und Geophysik der J. W. Goethe - Universität Frankfurt am Main, Theoretische Meteorologie, Robert-Mayer-Strasse 1, D-60325 Frankfurt am Main, Germany

With interest I have read the article by Kramm et al. (1995) to be referred to as KDL in the following. I wish to point to a critical matter in it which I believe deserves clarification for theoretical and practical reasons.

First, however, the problem in question will be briefly stated. In Appendix C (relevant to Section 2, p. 286) of the KDL paper the authors establish the following divergence expression (Eq. C3): $\nabla \cdot (\bar{\rho}\bar{v} + \overline{\rho' v'} - \bar{\rho}\langle v \rangle) = 0$ as a result of the equivalence of the Reynolds- and Hesselberg-averaged continuity equations. Without doubt one is here dealing with a trivial differential relation. It should not be regarded as a separate condition on the mass continuity equation. In reality, it rather roots in the general averaging formalism and is thus absolutely independent of a specific derivation operation.

To my surprise, KDL interpret this divergence expression (C3) as a differential constraint on the resulting momentum density vector $\bar{\rho}\bar{v} + \overline{\rho' v'} - \bar{\rho}\langle v \rangle$. And accordingly, they introduce a spatial vector, representing a constant of integration, which is made to vanish by assuming, as a boundary condition, that its components are zero at the surface of the Earth (see Eqs. (C4) and (C5) in KDL). Such a concept is basically erroneous. It is demonstrated in what follows, that in the crucial point the procedure presented by KDL is at variance with the elementary averaging technique. A relation between the Reynolds- and Hesselberg-averaged velocities (see (C5) in KDL) is fully described for arbitrary field functions $\chi(\mathbf{x}, t)$ by the universal and inherent rules of averaging. As a consequence (C5) is valid independently of a boundary-value condition and of (C3); the latter, therefore, turns out to be a meaningless triviality.

It is quite elementary to show the evidence of this. In the product integral $\overline{\rho\chi}$ the field variable $\chi(\mathbf{x}, t)$ has merely to be developed as a function of the density-weighted (Hesselberg) mean variable $\langle \chi \rangle$ or, alternatively, as a function of the simple (Reynolds) mean variable $\bar{\chi}$. It is quickest to carry out this procedure with the aid of the compact two-variable correlation principle (see KDL's equation (A9)) $\overline{\varphi\chi} = \bar{\varphi}\langle \chi \rangle + \overline{\varphi\chi''} = \bar{\varphi}\bar{\chi} + \overline{\varphi\chi'}$. By virtue of this and the assumption $\varphi = \rho$ (density)

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one is led directly to the desired relation

$$\langle \chi \rangle = \bar{\chi} + \frac{1}{\bar{\rho}} \overline{\rho' \chi'}, \tag{1}$$

which represents KDL's (C5)-equation in the particular case where $\chi = v$. Thus it is clear that the latter is to be understood as simple averaging technique algebra and not as something dependent upon a boundary-value problem.

Reference

Kramm, G., Dlugi, R. and Lenschow, D.H., 1995. A re-evaluation of the Webb correction using density-weighted averages. J. Hydrol., 166: 283–293.