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A note on Lettau's climatonomy equation and its use to classify droughts

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With 1 Figure

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Summary

It is argued in this note that Lettau's climatonomy equation that relates the flux balance equations of energy and water at the earth's surface to each other in a straight-forward manner can be used for classifying droughts. This climatonomy equation is derived, and the pros and cons are discussed in detail. It is emphasized that all relevant quantities can be obtained from meteorological and hydrological networks as well as radar and satellite observations.

1. Introduction

Several papers recently published in the *Bulletin of the American Meteorological Society* are dealing with the classification of drought by using so-called drought indices as characteristic measures (Heim, 2002; Keyantash and Dracup, 2002; Svoboda et al., 2002; Lawrimore et al., 2002). As pointed out by Heim (2002) in his review paper, various drought indices used in the United States during the last century are lacking in the attempt to classify drought. The new one, the so-called *Drought Monitor*, may be a suitable tool of tracking and displaying the magnitude and the spatial extent of drought and its impacts across the United States (Svoboda et al., 2002). From a physical and climatologic perspective, however, the characterization of drought demands measures more advanced than the current drought indices. Obviously, Lettau's (1969) *climatonomy equation* fulfils this requirement. It was not only applied by Lettau and various co-authors (e.g. Lettau and Baradas, 1973; Lettau and Lettau, 1975; Lettau and Hopkins, 1991; Lettau, 1994), but also by other climatologists like Dabberdt and Davis (1978), Flohn (1988), Lare and Nicholson (1990) as well as Nicholson and Lare (1990) to characterize arid regions. This equation can be considered as a result of interdisciplinary research already performed by Lettau three decades ago. Even though it relates the flux balance equations of energy and water at the earth's surface to each other in a straight-forward manner, it seems that Lettau's climatonomy equation is not well known in the community of meteorologists and hydrologists. Therefore, in the following it is derived and the pros and cons are discussed in detail. It is argued that all relevant quantities can be obtained from meteorological and hydrological networks as well as radar and satellite observations.

2. Physical background

Lettau's climatonomy equation combines the balance equations at the earth's surface of energy flux densities,

$$R_B - L_v(T_G)Q - H + G = 0, \tag{1}$$

and water flux densities,

$$P - R_O - Q - I = 0, \tag{2}$$

where a flux density (hereafter simply denoted as a flux) is counted positive when it is directed to the earth's surface. Here,

$$R_B = R_{S\downarrow}(1 - \alpha_S) + \varepsilon R_{L\downarrow} - \varepsilon \sigma {T_G}^4 \tag{3}$$

is the radiation balance, where $R_{S\downarrow}$ is the global radiation, α_S the albedo of the short-wave range, $R_{L\downarrow}$ is the incoming long-wave radiation emitted by the constituents of the atmosphere, $\varepsilon = 1 - \alpha_L$ is the absorptivity that is equal to the emissivity, α_L is the albedo of the long-wave range, and σ is the Stefan–Boltzmann constant. Furthermore, $L_v(T_G)$ is the latent heat of vaporization, often considered as dependent on the surface temperature, T_G. Moreover, Q and H are the fluxes of water vapor and sensible heat within the atmosphere, G is the soil heat flux, P is the precipitation, R_O is the surface runoff, and I is the infiltration.

Introducing the Bowen ratio, $Bo = H/(L_v(T_G)Q)$ into Eq. (1) and introducing the runoff ratio, $A_R = R_O/P$, as well as the infiltration ratio, $A_I = I/P$, into Eq. (2) yield the relations:

$$R_B - L_v(T_G)Q(1 + B_O) + G = 0, \tag{4}$$

and

$$P(1 - A_R - A_I) - Q = 0. \tag{5}$$

Note that most of these quantities can be measured by monitoring stations of a meteorological network and the lysimeter stations of a hydrological network. The radiation balance may also be derived from radiation measurements by satellites, as done, for instance, by Suomi (1958), Vonder Haar and Suomi (1971), Raschke et al. (1973) and many other authors (see, e.g. Kidder and Vonder Haar, 1995). Radar and satellite observations may also be used to estimate precipitation (e.g. Harrold, 1966; Collier, 1986; Kidder and Vonder Haar, 1995).

Lettau (1969) argued that for long-term considerations, the soil heat flux, G, and the infiltration, I, are of minor importance, and, hence, negligible. Following this argument, one obtains Lettau's climatonomy equation:

$$Bu = \frac{R_B}{L_v(T_G)P} = (1 - A_R)(1 + Bo). \tag{6}$$

Here, *Bu* is the dryness index (Budyko, 1958) also called the Budyko ratio. It relates the radiation balance to the portion of energy that is necessary to vaporize precipitation completely. Since the term $L_v(T_G)P$ is always positive, the dryness index has the same sign as the radiation balance. Equation (6), illustrated in Fig. 1, relates three characteristic non-dimensional numbers, namely *Bu*, *Bo*, and A_R. The resulting dryness index comprises meteorological, hydrological and some agricultural aspects. It may be considered as an illustrative example how the balance equations at the earth's surface of energy and water fluxes can be related to each other in the case of interdisciplinary research (see also Lettau and Baradas, 1973; Lettau and Lettau, 1975; Nicholson and Lare, 1990; Lettau and Hopkins, 1991).

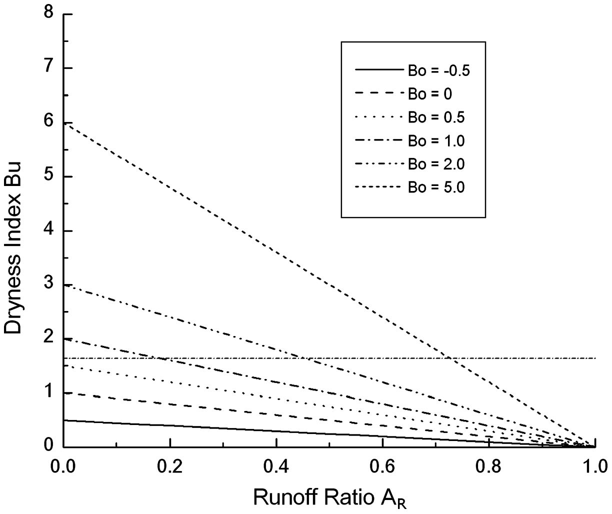

Fig. 1. Sketch of Lettau's climatonomy equation (with respect to Lettau's lectures held at the University of Wisconsin-Madison). The selected ranges suffice for the climatonomic interpretation of Budyko's (1958) illustration of tundra, forest, savanna, semi-desert, and desert climates (Lettau, 2002, personal communication). The dash–dot line parallel to the abscissa indicates that different effects may lead to the same *Bu*-results

Obviously, a variation in the runoff ratio and/or in the Bowen ratio results in a change in the dryness index. Following Lettau and Hopkins (1991), it can be expressed the best by the relative change of the dryness index,

$$\frac{\delta Bu}{Bu} = \frac{\delta R_B}{R_B} - \frac{\delta P}{P} = -\frac{\delta A_R}{1 - A_R} + \frac{\delta Bo}{1 + Bo}, \tag{7}$$

where $\delta R_B/R_B$ and $\delta P/P$ are the relative changes of the radiation balance and the precipitation, respectively. The applicability of Eqs. (6) and (7) requires that R_B, and P are non-zero fluxes and that the characteristic ratios $A_R \neq 1$ and $Bo \neq -1$. To ensure that these criteria can be fulfilled even for arid regions, Lettau's climatonomy equation should only be applied within the framework of long-term studies, where the dryness index might be characterized by the period under investigation.

3. The characterization of droughts

As pointed out by Flohn (1988), Lettau's climatonomy equation may serve as a suitable tool to characterize arid regions (i.e. drought). Charts of Bu-values, for instance, were plotted by Henning and Flohn and served as their contribution to the desertification conference in 1974 (as cited by Flohn, 1988).

It may be that for shorter periods (e.g. days to weeks) the assumption regarding the minor importance of G and I is not fulfilled exactly. Note that the quantity G is an appreciable energy flux on an hourly basis, but its net effect is relatively small when it is integrated over time periods of a week or more. For convenience, it may be expressed with the aid of the radiation balance according to

$$G = -\beta R_B, \tag{8}$$

where the proportionality factor β has been set, for instance, to $\beta \approx 0.1$ for a dry sandy soil or $\beta \approx 0.13$ for a moist bare soil (Oke, 1987). Recently, Liebethal and Foken (2006) investigated this relationship again. Based on their data collected during the LITFASS-2003 campaign that took place at one of the micrometeorological measurement sites near Lindenberg/Northeastern Germany from May 19 to June 17, 2003 (Beyrich, 2004), these authors found, on average, a proportionality factor of $\beta \approx 0.14$ for a loamy sand covered with 0.1 m tall maize shots at the beginning of the campaign which grew during the campaign reaching a stand height of up to 0.75 m. These authors also investigated the linear approach

$$G = -\beta R_B(t + \Delta t) - \delta \tag{9}$$

as suggested, for instance, by Fuchs and Hadas (1972) and Idso et al. (1975). Here, t is time and Δt is a time offset. Liebethal and Foken (2006) obtained from calibration following results: $\beta \approx 0.21$, $\delta \approx 28.1$, and $\Delta t \cong 1\,\text{h}$. As illustrated by these authors, the parameterization approach Eq. (9) follows the diurnal variation of the radiation balance in an acceptable manner. Whereas the parameterization approach Eq. (8) fails in the later afternoon and at night. Consequently, the latter has to be applied with care.

In summer, the daytime storage of energy slightly exceeds the nocturnal energy output and the soil gradually warms. The reverse is true in winter (Oke, 1987). Furthermore, instead of the runoff ratio, the infiltration ratio (or the sum of both ratios) might be applied. The use of the infiltration ratio seems to be reasonable for flat terrain, where surface runoff may become of minor importance. The infiltration can simply be derived from lysimeter registrations via the so-called lysimeter equation given by (e.g. Marshall et al., 1996; Mölders et al., 2003)

$$Q = P - G_R - \Delta S. \tag{10}$$

Herein, the water vapor flux Q is considered as a residuum that may represent the sum of the following processes: evaporation of soil water, puddles and intercepted water, transpiration by plants, and/or sublimation of snow. The infiltration is given by $I = G_R + \Delta S$, where G_R is the ground water recharge being assumed as the percolated water leaving the lysimeter at its outlet, and ΔS is the change in soil water storage being interpreted as the change in weight of the lysimeter during the registration interval. Usually, G_R, ΔS, and, in addition, precipitation, P, are routinely determined on a daily basis so that a change in weight by plant growth is usually negligible (e.g. Mölders et al., 2003). As often the number of lysimeter measurements is relatively small, improved land surface-hydrology modules coupled with regional atmospheric modeling systems (see, e.g. Pielke, 2002; Mölders and Walsh, 2004) could be applied to infer the observational data to those regions for which such data are unavailable. As shown, for instance, by Mölders et al. (2003), such land surface-hydrology modules are able to provide credible information, in particular, on surface runoff and infiltration as well as soil water availability and recharge, indispensable for agricultural purposes. Such a so-called inferential method is not new. Meyers et al. (1991) as well as Matt and Meyers (1993) applied comparable inferential measuring techniques to estimate the annual and seasonal dry deposition fluxes of sulfur dioxide, nitrate, and ozone on the basis of the NOAA-ATDD network data (see also NAPAP Newsletter, Vol. 2 (2), 1992).

Figure 1 also illustrates that different effects may lead to the same *Bu*-results. To avoid misinterpretation, Flohn (1988) proposed to plot not only *Bu* charts, but also charts of *Bo* and A_R values.

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